MODELING OF IRRIGATION AND DRAINAGE CANALS FOR CONTROLLER DESIGN

By J. Schuurmans,[1] A. J. Clemmens,[2] Member, ASCE, S. Dijkstra,[3] A. Hof,[4] and R. Brouwer[5]

ABSTRACT: Schuurmans et al. (1995a) presented a model for the design of water level controllers for irrigation and drainage canals that describes the essential characteristics of the processes relevant for canal control (such as water movements and control structures). This paper evaluates the accuracy of this model in two ways: (1) by comparing its frequency response with a model based on a finite difference approximation of the linearized St. Venant equations, and (2) by comparing simulation results with data from field experiments. Using these results, we characterize the accuracy of the model and discuss how these results can be taken into account in controller design.

INTRODUCTION

To perform their functions efficiently, irrigation and drainage canals must be managed appropriately; that is, the control structures need to be operated in such a way that the canals execute their functions adequately. In most cases, this requirement can be translated into a water level control problem.

The main function of an irrigation canal is to deliver water in an accurate and flexible way (Burt 1987). The delivery is said to be accurate if the actual supply matches the intended supply, and it is said to be flexible if the delivery meets the changing water requirements of the users. This main function can be translated into a water level control problem consisting of two parts.

First, the water levels in the canals located just upstream of the offtakes and control structures need to be controlled within a sufficiently small range. A prerequisite for allowing the flow rates through the offtakes and control structures to be maintained at any desired rate (that is, with sufficiently small variations around the desired rate) is that the water levels at the aforementioned locations are kept within a well-defined range.

Second, the water levels preferably should be controlled by adjusting the control structures located at the upstream ends of the reaches of the canal. This requirement guarantees that the delivery matches the demand; for instance, if a water level in one reach approaches the maximum allowable level, the inflow is apparently higher than necessary, and thus the inflow (the flow through the control structure at the upstream end of the reach) should be decreased. One control strategy that satisfies this latter requirement is the downstream control strategy (Burt 1987), in which a control structure is adjusted based on downstream information.

Several possibilities exist to realize a water level control strategy: (1) manual control, (2) automatic gates, and (3) automatic electronic devices.

[1]Res. Assoc., Univ. of Twente, Dept. of Electr. Engrg., P.O. Box 217, 7500 AE Enschede, The Netherlands. (Paper written when writer was with Delft Univ. of Technol., Dept. of Civ. Engrg., The Netherlands.)

[2]Supervisory Researcher and Hydr. Engr., U.S. Water Conservation Lab., 4331 East Broadway, Phoenix, AZ 85040.

[3]Asst. Prof., Dept. of Mech. Engrg., Delft Univ. of Technol., P.O. Box 5048, 2600 GA Delft, The Netherlands.

[4]PhD Candidate, Dept. of Civ. Engrg., Delft Univ. of Technol., P.O. Box 5048, 2600 GA Delft, The Netherlands.

[5]Prof., Dept. of Civ. Engrg., Delft Univ. of Technol., P.O. Box 5048, 2600 GA Delft, The Netherlands.

Note. Discussion open until May 1, 2000. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on October 30, 1997. This paper is part of the ***Journal of Irrigation and Drainage Engineering***, Vol. 125, No. 6, November/December, 1999. ©ASCE, ISSN 0733-9437/99/0006-0338–0344/$8.00 + $.50 per page. Paper No. 16867.

Under manual control, we include all control methods used by those who operate the structures, including remote manual control. Although operators are able to adapt to unforeseen changes in the behavior of the canal, their performance is limited by fatigue and loss of concentration, to compensate for which they need to work in shifts. Furthermore, one operator can operate a limited number of control structures at a time. As a result of these factors, the costs of operation are relatively high.

Gates that function as automatic water level controllers have been developed for the control of (1) the water level just upstream of the gate (for instance, weirs), (2) the water level just downstream of the gate, and (3) the difference in water level between the upstream and downstream sides of the gate (Ankum 1995). A control system that involves only gates is simple and reliable. However, the technical possibilities are limited, for only the water levels in the close vicinity of the gates can be controlled. Consider for instance the AVIS gates (Goussard 1987). The AVIS gate controls the water level just downstream of the gate within fixed limits. Therefore, a canal with AVIS gates can deliver water in an accurate and flexible way. However, the offtakes in existing canals are usually located at the downstream ends of the reaches, and the canal embankments are parallel to the bottom. In order to apply the AVIS gates, the offtakes need to be relocated at the upstream ends of the reaches and the embankments need to be raised. The costs involved in adapting an irrigation canal so that AVIS gates can be applied are relatively high in developed countries; application of these gates is therefore often not economically feasible (Burt 1987).

Since the 1960s, electronic devices have been used to control water levels. They offer the possibility of automating the operation of control structures without having to adapt the ‘‘hardware,’’ such as the canal embankments and offtake structures. In the following, we shall refer to the logic of electronic control devices as controllers.

Practicing control engineers of one company or institution usually apply one specific type of controller. Examples are ELFLO (Rogers et al. 1995), BIVAL (Chevereau and Schwartz-Bemeth 1987), and Dynamic Regulation (Rogier et al. 1987). The parameters of these controllers are usually tuned during operation or by trial and error on a simulation model. Furthermore, the design of these controllers has been mainly heuristic.

A systematic design method for controllers requires a suitable model of the processes most relevant for control. These include, among others, canal behavior (resulting from water movements), control structures, and actuators.

It is well known that the flow through a canal with control structures can be described quite accurately by the St. Venant equations in combination with algebraic discharge equations for the flow through the structures (such as offtakes and control gates). Many researchers propose controllers based on a nonlinear model of the canal system. Examples are methods based on solving the St. Venant equations backwards; that is, the controllers compute the control actions by inverting the discretized St. Venant equations (Wylie 1969; Falvey and Luning 1979; Chevereau 1991; and Liu et al. 1992). These controllers may show numerical instabilities, making them unreliable (Bautista et al. 1995). Apart from this problem, it is difficult to get insight into the influence of model parameter variations on closed loop behavior. To clarify this, consider the effect of an uncertain resistance coefficient (which is used in the St. Venant equations): How does the controlled system behave if this coefficient changes 10% from its assumed value? With a complicated nonlinear model, one is not able to answer this latter question until one has performed an extensive simulation analysis.

To obtain simple, reliable control systems, systematic design methods have been developed in linear control theory (Åström and Wittenmark 1984; Doyle et al. 1992). These design methods require a model that consists of linear ordinary differential equations or linear difference equations. It is clear that linearized canal models are by no means as accurate as the nonlinear models (based on the Saint Venant equations). However, it appears that in most control applications the model just needs to capture the ''essential'' dynamics in order to serve as a basis for the design of simple, reliable controllers. We believe that in many cases the essential canal dynamics can be captured by a simple linear model.

This paper evaluates the accuracy of a simplified model of the water movements presented by Schuurmans et al. (1995a). The results of these evaluations are useful for controller design. The paper is structured as follows: First, we briefly summarize the results described in Schuurmans et al. (1995a) and extend the water movements model with models of other relevant parts of a canal system: the control gates and signal converters. Then the accuracy of the model is evaluated from the response of a different model and field data. Using these results, we characterize how the model error depends on the model inputs and canal physics and briefly discuss how these results can be used for controller design. Finally, we present the conclusions.

SIMPLIFIED CANAL SYSTEM MODEL

Fig. 1 shows some of the most relevant processes for downstream control of a canal. The controller manipulates the actuators that move the control gates (i.e., a control action). Other structures may be used as well, but gates are most common. Sensors measure water levels and some of the load disturbances, such as offtake flows. The measured signals are sampled, that is, converted into digital signals by analog-to-digital (AD) converters before they enter the control system. The digital control signals from the controller are converted into analog signals by a digital-to-analog (DA) converter.

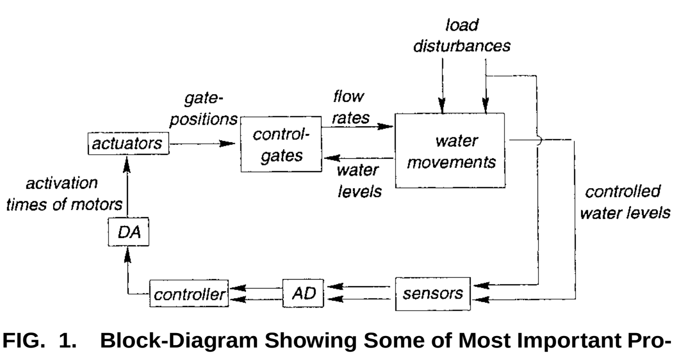

FIG. 1. Block-Diagram Showing Some of Most Important Processes in Controlled Canal

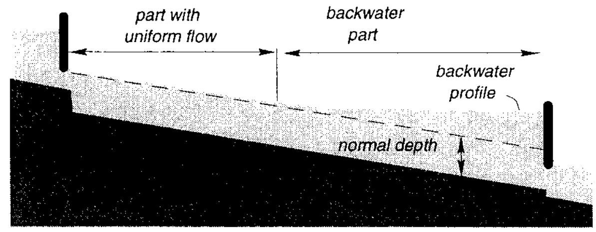

FIG. 2. Backwater Profile

In the following, we present control-oriented models for these processes.

Water Movements

Schuurmans et al. (1995a) analyzed and modeled the response of water levels to changes in flow (water movements). Here, we summarize the results described in that paper.

The water profile in controlled canals is usually a backwater profile, as shown in Fig. 2. We refer to the part of the reach affected by backwater as the backwater part, and the part unaffected by backwater as the part with uniform flow.

From the characteristics of the St. Venant equations, it is known that a disturbance, created somewhere in a reach, results in two wave movements. Defining c as the critical celerity and V as the mean velocity, one wave travels with velocity $V + c$ and one travels with velocity $V - c$ (Abbott 1979; French 1986). During travel, the waves deform. Waves that travel with velocity $V - c$ damp essentially after traveling over a certain distance. How quickly waves deform depends on the flow conditions. In parts with uniform flow (where the depth of flow is practically at normal depth), the waves deform considerably faster than in backwater-affected parts. An explanation for this is that in the backwater part the mean velocity is relatively small, and thus there is less friction from the channel boundary; according to the St. Venant equations, the friction from the channel boundary is the only cause for damping.

When a wave arrives at a boundary (a control structure, for instance), part of the wave is reflected. As a consequence, in leveled reaches that are sufficiently short, waves can travel up and down several times before they damp out. If in such a reach (with length L) a wave is initiated from one of the boundaries, it returns after a period (T_r), which can be approximated by

$$T_r \approx \frac{L}{V + c} + \frac{L}{c - V} \tag{1}$$

Suppose that the flow rate at the upstream boundary ($Q(0, t)$) would vary in a sinusoidal way with period T_r, while the flow rate at the downstream boundary ($Q(L, t)$) would remain constant, for instance, according to

$$Q(0, t) = \sin\left(\frac{2k\pi}{T_r} t\right) + Q_0, \quad Q(l, t) = Q_0 \tag{2}$$

with Q_0 the initial flow rate. Then waves would add to each other, causing a standing wave, or, in other words, resonance. The resonance frequencies (in radians) can be approximated by

$$\omega_r(k) = \frac{2\pi k}{T_r}, \quad k = 1, 2, 3, \ldots \tag{3}$$

Hence, there are infinitely many resonance frequencies, each a multiple of the lowest one.

Finite Difference Model

One way to obtain a linear model of the water movements is to approximate the partial differentials in the linearized St.

Venant equations by finite differences, whereby the canal is divided into nodes and branches, and for each branch the differentials are approximated by finite differentials. This modeling technique is described in more detail in Balogun et al. (1988); Reddy (1990); and Malaterre (1994). We shall refer to such a model as a finite difference (FD) model. If the number of nodes chosen is sufficiently high, the FD model can be very accurate for ''small'' variations of its inputs around their steady-state values. However, the FD model contains many parameters whose importance is unclear, such as how the model's response changes with changing model parameters. (A parameter is defined here as a physical parameter, such as Manning's coefficient, the length of a pool, or its bed slope.) For instance, the FD model contains Manning's coefficient, which in leveled reaches can often be changed 100% without affecting the model's largest time constant (its main dynamics). Manning's coefficient does change the travel time of gravity waves, but in leveled reaches these waves hardly affect the largest time constant. The model described in Balogun et al. (1988) is an example. It already contains 18 parameters when four nodes are used. Therefore, only the main characteristics of the dynamics of water movements were identified and described by the model presented in Schuurmans et al. (1995a).

Integrator Delay Model

In the backwater part, the dynamics are complicated: waves move up and down and reflect against the boundaries. However, at low frequencies, the water level ''integrates'' flow variations in the backwater part. In other words, the backwater part can be considered to behave as an integrator or reservoir for ''low'' frequencies. Referring to Fig. 3, the dynamics in the backwater part are approximated by

$$A\,\frac{dh(x,\,t)}{dt} = q(L_u,\,t) - q(L,\,t) \tag{4}$$

where h is the variation of water level or depth of flow around steady state (m), q the variation of flow rate around steady state (m^3/s), A the value of surface area of the backwater part (m^2), L_u the length of the part with uniform flow (m), x the distance from the upstream boundary, and t the time (s). Hereafter, we shall speak of ''the water level h'' and ''the flow rate q'' to refer to variations of these variables around their steady-state values.

In the part with uniform flow, waves with velocity $V - c$ dampen relatively quickly (Schuurmans et al. 1995a). Therefore, the influence of these waves is neglected, and it is assumed that only upstream flow changes can influence the flow. To describe the effect of the upstream inflow $q(0,\,t)$ on the downstream flow in a part with uniform flow, a ''pure'' delay model is assumed, as follows:

$$q(x,\,t) = q(0,\,t - T(x)) \tag{5}$$

with $T(x)$ given by Schuurmans et al. (1995a):

$$T(x) = \frac{2x}{(1 + \kappa)V_0} \tag{6}$$

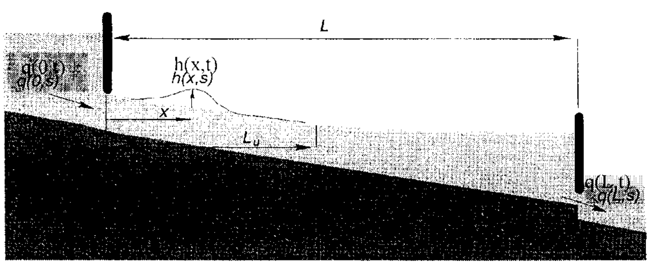

FIG. 3. Meaning of Notations

The parameter κ is given by

$$\kappa = 1 + \frac{4}{3}\,\frac{P_0}{B_0}\,\frac{dR}{dY}\bigg|_0 \tag{7}$$

where B is the flow top width (m), P the wetted perimeter (m), R the hydraulic radius (m), and Y the depth of flow (m). The subscript zero refers to steady-state conditions.

Eqs. (5) and (6) ''assume'' that waves travel with velocity $V_0(1 + \kappa)/2$. In many cases, this velocity is 1–2 times the mean velocity V_0, which is usually considerably lower than $V_0 + c_0$. Hence, the effect of wave deformation is taken into account by assuming a ''lower'' velocity of the wave than the theoretical velocity $V_0 + c_0$.

If the whole reach is affected by backwater ($L_u = 0$), the model given by (4) can be applied. If the backwater affects only part of the reach, the flow $q(L_u,\,t)$ is related to the upstream inflow $q(0,\,t)$ via (5). Hence

$$q(L_u,\,t) = q(0,\,t - T(L_u))$$

This means that the inflow at $x = L_u$ at time t is taken to be equivalent to the inflow $q(0,\,t)$, delayed with $T(L_u)$ time units. From here on, we shall ignore the dependence of T on x and speak of T instead of $T(L_u)$.

The parameters of this model (A and T) can be computed rather accurately from the geometry of the channel and the steady-state values of the flow rate(s) and water depths. If the geometry changes over the distance, it may be necessary to compute the parameters (A and T) for small parts and sum them thereafter. If geometrical data are lacking or hard to obtain, it may be more practical to obtain the parameters by fitting the model's response to measurements: using measurements of the flow rates at the boundaries, the model's response can be fitted to measured water levels by adjusting the parameters.

Control Structures and Actuators

The flow through control structures can usually be described quite accurately by an algebraic nonlinear discharge relation, based on the Bernoulli equation and a mass balance; the discharge relations can often be described as

$$Q = Q(U_g,\,H_1,\,H_2) \tag{8}$$

where U_g is the adjustable variable, such as the position of a gate, H_1 the water level upstream of the structure, H_2 the water level downstream of the structure, and Q the flow rate (m^3/s). Linearization of this equation for a change in flow rate q gives

$$q = c_g u_g + c_1 h_1 + c_2 h_2 \tag{9}$$

with h_1, h_2, q and u_g the variations of H_1, H_2, Q, and U_g around their steady-state values, respectively, and with c_g, c_1, and c_2 given by

$$c_g = \left(\frac{\partial Q}{\partial U_g}\right)_0; \quad c_1 = \left(\frac{\partial Q}{\partial H_1}\right)_0; \quad c_2 = \left(\frac{\partial Q}{\partial H_2}\right)_0 \tag{10a–c}$$

The water levels h_1 and h_2 can be obtained as outputs of a water movement model, while q is an input for the water movement model.

Although there are many types of actuators, we consider one of the most common actuators here: an electromotor drives the position (U_g) of the control gate with a constant speed (V_m) during a desired time (a_m). This may seem to limit the possibilities of control, but the gate speed can be pulse modulated; keeping this in mind, the speed of gate movement can be approximated by Papageorgiou and Messmer (1985):

$$\frac{dU_g}{dt} \approx V_m a_m, \quad |a_m| \le 1 \tag{11}$$

This model shows a nonlinearity, since there is a maximum gate speed ($|a_m| \leq 1$). In reality, the relation from the activation time (a_m) to the position (U_g) shows even more nonlinearities. First, there is a minimal gate movement or dead zone to avoid inexecutable small gate changes. Second, there is hysteresis: If the actuator reverses movement, the gate movement does not immediately ''follow.'' This can be caused by loose connections in the actuator (i.e., gear lash) or friction between the gate and the construction. Third, the gate can be moved within a limited range only.

For controller design, the actuator model as given above can be used, but without the limitation on the activation time.

Converters

The DA conversion is usually performed by a zero-order-hold (ZOH) mechanism. In this mechanism, the digital signal is held constant between two sample times. The modeling of a continuous time system with a ZOH on the inputs and a sampler on the output is done most naturally in the discrete time domain. There exists a method, known as ZOH sampling, to transform continuous time linear models (with a ZOH and a sampler) into discrete-time models (Åström et al. 1984).

Here we present the result of this transform, applied to the model of the water movements, given by (4) and (5). If the delay time T is exactly an integer τ times the sample period T_s, it can be shown that ZOH sampling of this model gives

$$\Delta h(k) = \frac{T_s}{A}[q_1(k-\tau) - q_2(k)] \tag{12}$$

with k the discrete time level ($k = 0, 1, 2, \ldots$), $\Delta h(k) = h(k) - h(k-1)$ the water level change (m), q_1 and q_2 the inflow and outflow of the pool, respectively, and $\tau = T/T_s$ the discrete time delay. If the delay τ is not precisely an integer times the sample period T_s, the discrete time model is slightly different. However, (12) can be used as an approximation whereby the parameter τ is obtained by rounding off the quotient T/T_{sx} to the nearest integer.

EVALUATION OF ACCURACY

In this section we evaluate the accuracy of the model, as described above, whereby we focus on the integrator delay (ID) model. We first derive an ID model for a hypothetical canal and compare its frequency response to that of an FD model. The flow in this canal is subcritical, allowing us to evaluate the accuracy of the ID model in ''normal'' circumstances. In particular, we investigate the resonance phenomena occurring in this canal in detail. Then we derive a model for an irrigation canal in the Maricopa Stanfield Irrigation and Drainage District in Arizona and compare this model's response with field measurements.

Linear Model of Test Canal One

Test Canal One is a hypothetical irrigation canal, defined by the ASCE Task Committee on Canal Control Algorithms in Kacerek et al. (1995). Although the actual test canal consists of eight reaches, we consider only the two upstream reaches here. The reason for choosing these two reaches is that these cover two different possibilities. Reach one is completely affected by backwater, whereas reach two has a part with uniform flow, as shown in Fig. 4. In this figure, the variables h_1 and h_2 are water levels, q_1 and q_2 are the flow rates through the control gates, and d_1 and d_2 are offtake flow rates; recall that all variables denote variations around their steady-state value. The geometrical data and other relevant parameters of the two reaches are given in Table 1.

In this example, we formulate an integrator-delay (ID) model and a finite difference (FD) model for the two upstream reaches of Test Canal One and compare the frequency responses of these models, whereby each model describes the relation between the flow rates q_1 and q_2 and the water levels h_1 and h_2.

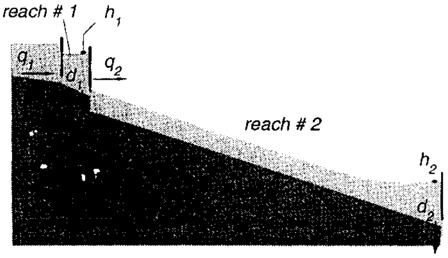

FIG. 4. Longitudinal View of Reaches One and Two during Initial Conditions. Vertical Scale is Different from Horizontal Scale to Show Details

TABLE 1. Parameter Values of Test Canal One

Length of reaches, *L* (1)	100 m (reach # 1), 1,200 m (reach # 2) (2)
Manning's resistance coefficient, *n*	0.014 s/m$^{1/3}$
Bed slope, *I*	0.002 m/m
Bottom width, B_b	1.0 m
Side slope, ν	1.5 horizontal/vertical

Note: Cross-sections are trapezoidal.

The models were derived for a steady-state condition in which the flow rate was equal to 0.8 m^3/s, while the downstream water levels were at target level (a depth of 0.9 m).

ID Model

The continuous time ID model is given by

$$A_i \frac{dh_i(t)}{dt} = q_i(t - T_i) - q_{i+1}(t) - d_i(t) \tag{13}$$

where A_i is the backwater surface of reach i and T_i the delay time for reach i. These parameters were computed as follows: The water profile in the backwater part was assumed to be given by a horizontal line, while the water profile in the part with uniform flow was assumed to be parallel to the bottom at a depth equal to the normal depth of flow. These assumptions are reasonable for this particular canal, as can be seen in Fig. 4: the water profiles satisfy the description as given above rather well. The backwater surface is then determined over the part from the downstream end up to the point where the two lines intersect. The delay time was computed with (6) using the length of the part with uniform flow for x and the normal depth to compute the mean velocity of flow V and the parameter κ. This leads to $A_1 \approx 340$ m^2, $A_2 \approx 756$ m^2, $T_1 = 0$ s, and $T_2 = 528$ s. Here, the subscript refers to the reach number.

FD Model

The FD model was obtained using the method described in Balogun et al. (1988), whereby each reach was divided into 20 nodes.

Comparison of Frequency Responses

Fig. 5 shows Bode diagrams of the models; the titles ''q_i –> h_j'' indicate the relation from the flow rate (q_i) through the ith gate to the water level (h_j) in the jth reach. The Bode diagram gives detailed information on the steady response (which occurs after the transient response) of a system to sinusoidal signals; since each periodic signal can be written as

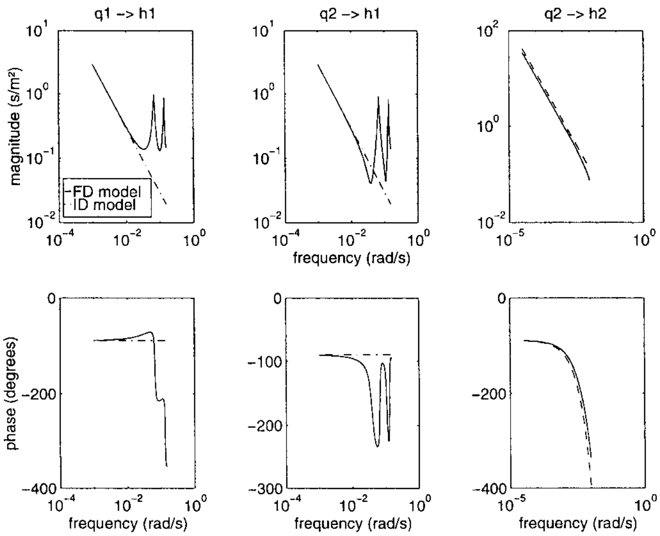

FIG. 5. Bode Diagrams of Test Canal One

a sum of sinuses (the Fourier series), the Bode diagram provides full information on the steady response to any periodic signal. For instance, for the relation between q_1 and h_1, at a frequency of 0.03 rad/s, the magnitude is (according to Fig. 5) approximately 0.15 s/m^2, and the phase is approximately $+8°$ ($\approx$0.14 rad). This means that if $q_1(t)$ would vary according to $q_1(t) = \sin(0.03t)$, the water level would show the steady response (after the transient response): $h_1(t) = 0.15 \sin(0.03t + 0.14)$.

From the FD model's response, resonance can be observed in reach one. The resonance peak gain (R_p) is approximately 1 s/m^2, while the resonance frequencies are approximately 0.07, 0.14, 0.21, . . . rad/s [0.07*k rad/s (k = 1, 2, 3, . . .)]. These resonance frequencies are in good agreement with the frequencies predicted by (3). To show this, we first compute the cross-sectional area (A_c) and the width at surface level (B), assuming the depth is everywhere equal to the target depth (Y_t) of 0.9 m (using data from Table 1):

$$B = B_b + 2\nu Y_t \approx 3.7 \text{ m}, \; A_c = (B_b + \nu Y_t)Y_t \approx 2.12 \text{ m}^2.$$

It follows that $c_0 = \sqrt{(gA_c/B)} \approx 2.37$ m/s and $V_0 = Q/A_c \approx 0.38$ m/s. Since the length (L) of reach one is 100 m, (3) gives

$$\omega_r(k) \approx \frac{2\pi k}{\dfrac{L}{V_0 + c} + \dfrac{L}{c - V_0}} \approx \frac{2\pi k}{\dfrac{100}{0.38 + 2.37} + \dfrac{100}{2.37 - 0.38}}$$

$$\approx 0.07k(\text{rad/s}) \tag{14}$$

For reach one, where the backwater extends over the entire length, the ID model is inaccurate in the neighborhhood of the resonance frequencies, but is rather accurate for lower frequencies. This indicates that the main dynamics (the low frequent dynamics) in leveled reaches are indeed affected by the backwater surface only. This explains, for instance, why Manning's coefficient in the FD model can be changed 100% without affecting the main dynamics. In reach two, a large portion is affected by uniform flow; consequently, no resonance can be observed in the relation between q_2 and h_2. In this reach, the ID model is rather accurate over all frequencies. This again shows that the main canal dynamics can be described by a much simpler model than the FD model.

Due to the simplicity of the ID model, it is also possible to get a grip on the nonlinear effects of the ''true'' dynamics. For instance, the dynamics (at low frequencies) of a leveled canal pool with a square canal cross section can be predicted rather accurately by the ID model, since the backwater surface is constant in such a pool. For pools with trapezoidal cross sections (which usually is the case), the ID model error can be expected to increase if the backwater surface A differs more from its nominal value (the value that is assumed in the ID model). When it concerns a sloping reach, the effects of the nonlinearities are even more pronounced. For instance, the delay time T can then easily vary 100%. Although in such cases the ID model is by no means as accurate as a nonlinear model, it can still be useful for controller design. For instance, it is possible to link the variations of the flow rate with the delay time T and the backwater surface A. By doing so, one obtains a quasilinear model that can be used as a basis for an adaptive controller, that is, a controller whose settings change with changing flow rate. Alternatively, the model errors can be quantified and used in the design of a so-called robust controller, that is, a controller that is stable and performs adequately in the face of model errors. Details on the design of adaptive and robust controllers (using the ID model) can be found in Schuurmans (1997). For a practical of these techniques, see Schuurmans et al. (1997).

So far, the accuracy of the ID model has been evaluated by comparing its response to that of the FD model. In the next section, the accuracy of the model is evaluated using field data.

Experimental Model Verification on WM Canal

The accuracy of a model containing an integrator delay model for the water movements has been verified experimentally on a lateral irrigation canal in the Maricopa Stanfield Irrigation and Drainage District in Arizona. This canal consists of eight reaches, with lengths varying from 100 to 2 km. Fig. 6 shows a longitudinal view of this canal. The graph is to scale, although the vertical scale is different from the horizontal scale to show details.

The WM Canal takes water from the Santa Rosa canal. Offtake structures are located at the downstream ends of the reaches. Therefore, the water level at the downstream end of each reach is controlled. The canal is currently operated manually, but there are plans to operate it automatically (Clemmens et al. 1995). In fact, several closed-loop experiments have been conducted on this canal; the tests and results of these tests are described in Schuurmans and Liem (1995b) and Liem (1994).

Linear Model of WM Canal

Inspection of the water profiles in the WM Canal, as shown in Fig. 6, leads to the conclusion that all reaches but the first one have a part with uniform flow. In fact, flow is supercritical in the parts with uniform flow and shows a hydraulic jump when entering the backwater part. The integrator delay model for this particular canal becomes

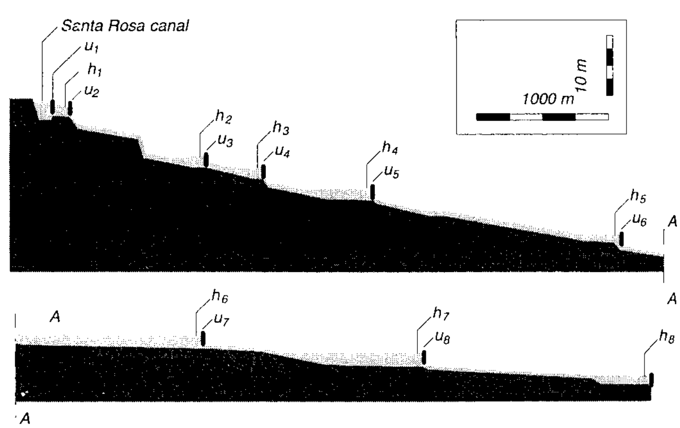

FIG. 6. Longitudinal View of WM Canal

$$A_i \frac{dh_i(t)}{dt} = q_i(t - T_i) - q_{i+1}(t) \quad (15)$$

where h_i is the variation of water level number i (m), q_i the variation of flow rate through gate i (m^3/s), A_i the backwater surface of reach i (m^2), and T_i the delay time of reach i (s). The offtake structures are not modeled here, as they were not used at the time the experiments were conducted. The control gates were orifices. The flow through these (sharp-crested) gates was modeled by linearization of

$$Q = B\mu U_g \sqrt{2g(H_1 - \mu U_g} \quad (16)$$

with Q is the flow rate (m^3/s), B the width under the gate (m), μ the contraction coefficient, U_g the opening height under the gate, and H_1 the upstream water level. For gate i, the linearized discharge equations can then be written as

$$q = c_{g,i}u_i + c_{1,i}h_i \quad (17)$$

where u_i is the variation of the opening of gate number i (m) and $c_{g,i}$ and $c_{1,i}$ are constants that follow from linearization and the initial values of flow variables, such as the flow rate Q_0.

The physical parameters, such as the gate widths and resistance coefficient, were gathered from design drawings. Using this data, the parameters A_i were determined by assuming the water profile in the backwater part to be horizontal. The parameters T_i were computed using (6), where x was taken as the length of the uniform part. (All physical parameters of the WM canal (geometrical data, roughness coefficients, etc.), as well as the computer code to generate the ID model parameters A_i and T_i from these data, can be obtained via electronic mail by sending an email to a.hof@ct.tudelft.nl.) For sharp-crested orifices, the contraction coefficient (μ) is usually close to 0.63 (Bos 1976); therefore, we used μ = 0.63.

Experiments and Results

To test the accuracy of the model, five experiments were conducted that are described here and in Ellerbeck (1995). In each of these experiments, the openings for one or two gates were changed while the variations of the water levels in two adjacent reaches were recorded, whereby the gate changes were made small to avoid nonlinear effects. For comparison, we simulated the response of the water levels using the model as described above, whereby the model parameters were computed for the steady-state conditions as recorded at the beginning of the experiment.

Fig. 7 shows the gate opening variations around their initial values ($u1 \ldots u8$) and the water level variations ($h1 \ldots h8$) around the initial levels. The variables u and h are located as shown in Fig. 6: $h1$ is the water level (deviation) in the first reach, $u1$ is the gate opening (change) of the most upstream gate, and so on. The water level responses of the linear model are plotted as solid lines, and the measured water level responses of the WM Canal are plotted as dots. Two graphic windows next to each other belong to one experiment. Hence, the upper graphs belong to one experiment, the next two to another experiment, and so on. In the first experiment, gate $u2$ was changed, followed by a change to gate $u3$. Both of these changes affected water level $h2$, but only gate $u2$ affected water level $h1$. The observed water levels are well predicted by this linear model.

For the remaining experiments, only one gate change was made, which affects the water levels both upstream and downstream. It can be seen that the response of the linear model is reasonably accurate, apart from the simulated water level $h5$, which deviates considerably from the measured level after approximately 75 min. The difference resulted from water flow over the weirs of the check structure, which was not modeled.

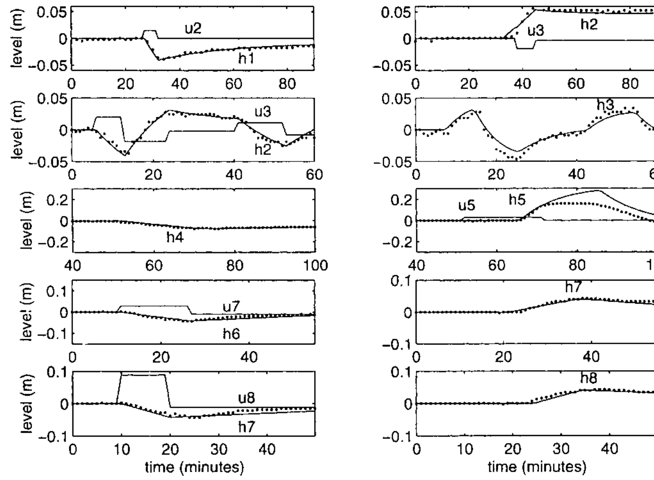

FIG. 7. Water Level Variations (h1–h8) in Response to Gate Opening Variations (u1–u8). Solid Line: Simulated Levels; Dots: Measured Levels

These weirs protect the canal from overtopping by passing excess flows downstream. The results of these experiments show that the ID model is accurate if the input changes are relatively small. Indeed, the experiments were performed in such a way that the input changes were sufficiently small. In practice, the input changes may not be small, and therefore the model's response may differ substantially from the actual response. As we stated before, these errors can be reduced by adapting the model parameters on-line, or, alternatively, the model error can be quantified and used to design a robust controller. Furthermore, the model error can be reduced by manipulating the flow rates through the control gates as control inputs instead of the gate positions. By doing so, the nonlinearities introduced by the control gates are no longer part of the model. To achieve this, low-level flow rate controllers can be used to manipulate the control gates so as to maintain the flow rates at the desired levels, as in Schuurmans (1997a, 1997b).

CONCLUSIONS

For the design of water level controllers in irrigation and drainage canals, a simple linear model (the ID model) has been presented in Schuurmans et al. (1995a). This paper has analyzed the accuracy of this model and discussed how the inaccuracies can be taken into account in controller design.

For a hypothetical canal, the accuracy of the ID model was evaluated using a finite difference (FD) model as a reference. It appeared that, when resonance occurs, the ID model's accuracy is limited up to the first resonance frequency. Other than that, the frequency responses of both models match well.

The accuracy of the ID model was further verified successfully using field measurements of an irrigation canal in Arizona. However, the input variations were deliberately kept small to avoid nonlinear effects.

In practical situations, the input variations may not be small, and the model's response will differ from the actual response: nonlinearities are not modeled by the ID model (nor by the FD model). There are two ways in which the nonlinearities can be taken into account in controller design. First, it is possible to update the model and control parameters on-line; this results in an adaptive controller. Second, it is possible to quantify the model errors and use these in connection with the ID model to design a robust controller.

In conclusion, the model is rather accurate under the conditions that (1) input variations are small (around one steady-state value), and (2) (in leveled reaches) the frequency contents of the input are concentrated below the (lowest) resonance frequency.

ACKNOWLEDGMENTS

We would like to thank the water master G. Sloan for this cooperation and assistance during the experiments on the WM canal in the Maricopa Stanfield Irrigation and Drainage District. Furthermore, we would like to thank the following organizations for their financial support: the U.S. Water Conservation Lab, the Netherlands Organization for Scientific Research (NWO), the Delft University of Technology (Universiteitsfond Delft and College van Bestuur) and the Royal Dutch Institute of Engineers (KIvI).

APPENDIX. REFERENCES

Abbott, M. B. (1979). *Computational hydraulics: Elements of the theory of free surface flows*. Pitman Publishing Limited, London.

Ankum, P. (1995). ''Flow control in irrigation and drainage.'' *Rep. No. 65*, Water Mgmt. Dept., Delft, The Netherlands.

Åström K. J., and Wittenmark, B. (1984). *Computer controlled systems: Theory and design*. Prentice-Hall International, London.

Balogun, D., Hubbard, O. S., and De Vries, J. J. (1988). ''Automatic control of canal flow using linear quadratic regulator theory.'' *J. Hydr. Engrg*., ASCE, 114(1), 75–102.

Bautista, E., Clemmens, A. J., Strelkoff, T. S. (1995). ''Inverse computational methods for open-channel control.'' *Proc., First Int. Conf. on Water Resour. Engrg*., San Antonio, Tex.

Bos, M. G. (1976). ''Discharge measurement structures,'' *Rep. No. 4*, Lab. for Hydr. and Catchment Hydrol., Wageningen, The Netherlands.

Burt, C. M. (1987). ''Overview of canal control concepts.'' *Proc., Planning, Operation, Rehabilitation and Automation of Irrig. Water Delivery Sys*., ASCE, New York.

Chevereau, G., and Schwartz-Bemeth, S. (1987). ''BIVAL system for downstream control.'' *Proc., Planning, Operation, Rehabilitation and Automation of Irrigation Water Delivery Sys*., ASCE, New York.

Chevereau, G. (1991). ''Contribution à l'etude de la regulation dans les systemes hydrauliques à surface libre,'' PhD thesis, Institut National Polytechnique de Grenoble, Grenoble, France.

Clemmens, A. J., Strand, R. J., and Schuurmans, J. (1995). ''Application of automatic control of MSIDD WM lateral canal.'' *Proc., First Int. Conf. on Water Resour. Engrg*., San Antonio, Tex.

Doyle, J. C., Francis, B. A., and Tannenbaum, A. R. (1992). *Feedback control theory*. Macmillan, New York.

Ellerbeck, M. B. (1994). ''Model predictive control for an irrigation canal,'' MSc thesis, Lab. for Measurement and Control, Delft, The Netherlands.

Falvey, H. T., and Luning, P. C. (1979). ''Gate stroking.'' *Rep. REC-ERC-79-7*, U.S. Bureau of Reclamation, Washington, D.C.

French, R. H. (1986). *Open-channel hydraulics*. McGraw-Hill, New York.

Goussard, J. (1987). ''NEYRTEC automatic equipment for irrigation canals.'' *Proc., Planning, Operation, Rehabilitation and Automation of Irrigation Water Delivery Sys*., ASCE, New York.

Kacerek, T., Clemmens, A. J., and Sanfilippo, F. (1995). ''Test cases for canal control algorithms with examples.'' *Proc., First Int. Conf. on Water Resour. Engrg*., San Antonio, Tex.

Liem, G. R. (1994). ''Controller design for irrigation canals,'' MSc thesis, Lab. for Measurement and Control, Delft University of Technology, Delft, The Netherlands.

Liu, F., Feyen, J., and Berlamont, J. (1992). ''Computational method for regulating unsteady flow in open channels.'' *J. Irrig. and Drain. Engrg*., ASCE, 118(10), 674–689.

Malaterre, P. O. (1994). ''Modélisation, analyse et commande optimale LQR d'run canal d'irrigation,'' PhD thesis, Ecole Nationale du Génie Rural des Eaux et des Forêts (ENGREF), Paris.

Papageorgiou, M., and Messmer, A. (1995). ''A continuous-time and discrete-time design of water flow and water level regulators.'' *Automatica 1985*, 21(6), 649–661.

Reddy, J. M. (1990). ''Local optimal control of irrigation canals.'' *J. Irrig. and Drain. Engrg*., ASCE, 116(5), 616–631.

Rogers, D. C., et al. (1995). *Canal systems automation manual*, Vol. 2, 1st Ed., U.S. Dept. of the Interior, Bureau of Reclamation, Denver, Colo.

Rogier, D., Coeuret, C., and Brémond, J. (1987). ''Dynamic regulation on the Canal de Provence.'' *Proc., Planning, Operation, Rehabilitation and Automation of Irrig. Water Delivery Sys*., ASCE, New York.

Schuurmans, J. (1997). ''Control of water levels in open-channels,'' PhD thesis, Dept. of Civ. Engrg., Delft University of Technol., Delft, The Netherlands.

Schuurmans, J., Bosgra, O. H., and Brouwer, R. (1995a). ''Open-channel flow model approximation for controller design.'' *Appl. Math. Modelling*, 19, 525–530.

Schuurmans, J., and Liem, G. (1995b). ''Controller design for the WM Canal.'' *Proc., First Int. Conf. on Water Resour Engrg*., San Antonio, Tex.

Schuurmans, J., Schuurmans, W., Berger, H., Meulenberg, M., and Brouwer, R. (1997). ''Control of water levels in the Meuse river.'' *J. Irrig. and Drainage Div*., ASCE, 123(3), 180–184.

Wylie, E. B. (1969). ''Control of transient free-surface flow.'' *J. Hydr. Div*., ASCE, 95(1), 347–361.